



HYPOTHETICAL LEARNING TRAJECTORY IN LEARNING THREE-VARIABLE LINEAR EQUATION SYSTEMS

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ABSTRACT

The purpose of this research is to develop a learning trajectory for the topic of systems of linear equations in three variables, known as a Hypothetical Learning Trajectory (HLT), based on the identification of learning barriers to help address the difficulties faced by students. The research was conducted at SMK Negeri 1 Padoherang, with the subjects comprising 28 class X students who had studied the Three-Variable Linear Equation Systems material and one mathematics teacher. The data collection methods included observation, interviews with the mathematics teacher, diagnostic tests, and literature reviews. The results of this study showed that students faced difficulties due to a lack of understanding of prerequisite material, difficulty in determining the correct mathematical model, and difficulty in comprehending word problems involving systems of linear equations with three variables. The findings indicate the need for a more structured and guided approach to teaching SPLTV concepts so that students can understand and apply variables correctly. The implementation of HLT can help improve students' mathematical modeling skills, enabling them to be more confident and competent in handling mathematical problems. Thus, this study provides a significant contribution to improving the effectiveness of mathematics education by addressing students' encountered learning obstacles.

Keywords: Learning Obstacle, Didactical Design, Three-variables Linear Equation System

INTRODUCTION

Mathematics is an integral part of the educational curriculum due to its relevance in everyday life. Hasibuan (2015) emphasizes the importance of practice in understanding mathematical concepts; however, students often neglect prerequisite materials, leading to anxiety when facing mathematical problems. Kesumawati & Aulia (2017) add that mastering mathematics not only requires critical, logical, and systematic thinking but also precision in understanding concepts. The 2013 curriculum underscores the importance of understanding mathematical concepts as a foundation for solving daily problems. Kesumawati (2008) highlights the role of conceptual understanding in the learning process, while Purnama, Nursalam, & Sulasteri (in Rahayu & Fuadiah, 2021) stress the need for understanding prerequisite materials before tackling more complex mathematical topics.

According to Brousseau (in Septiana, Kesumawati, & Fuadiah, 2021), there are three factors that cause students to struggle with understanding lesson materials: ontogenic, didactic, and epistemological

obstacles. To investigate these obstacles, researchers conducted an initial study by administering tests to 28 students in class X at SMK Negeri 1 Padaherang. The analysis results showed that students faced difficulties due to a lack of understanding of prerequisite material, difficulty in determining the correct mathematical model, and difficulty in comprehending word problems involving systems of linear equations in three variables.

Previous research conducted by Sari (2023) discussed the use of didactic situations in overcoming epistemological obstacles in the material of linear equations in two variables. The results of this study indicate that the didactic approach can help students understand the correct mathematical model. Additionally, Wicaksono (2024) also identified learning obstacles in the material of linear equations in one variable. This research found that ontogenic, didactic, and epistemological obstacles are the main factors hindering students' understanding. Therefore, this research supports the findings that these obstacles need to be addressed through an appropriate didactic approach.

Furthermore, Nurhasanah (2019) in her research revealed that well-designed didactic designs can minimize the epistemological and ontogenic barriers experienced by students, as well as improve their mathematical problem-solving abilities. Similarly, Muthmainnah (2023) showed that students' understanding of the concept of two-variable linear equations can be enhanced by providing more varied and engaging material, addressing the limitations of reference sources that often become obstacles in learning.

Additionally, research conducted by Pratomawati (2020) also supports these findings by developing a didactic design to overcome students' learning barriers in inverse function material, where the design was able to improve students' understanding through more structured and comprehensive teaching strategies. Meanwhile, Maarif, Setiarini, & Nurafni (2020) in their analysis of epistemological barriers in the two-variable linear equation system indicated that an approach that takes into account students' epistemological needs can significantly improve their understanding.

However, until now, research discussing the Hypothetical Learning Trajectory (HLT) in the learning of the Three-Variable Linear Equation System (SPLTV) has not yet been found. Existing studies, such as those by Hidayati, Deciku, & Azizah (2022), have focused more on SPLDV (Two-Variable Linear Equation System) with the Realistic Mathematics Education (RME) approach, which has shown positive results in overcoming students' learning barriers. Another study by Septiana, Kesumawati, & Fuadiah (2021) that developed HLT on three-variable linear equation system material through assessment-oriented also focused more on general teaching methodologies without specifically addressing SPLTV. Therefore, further research on HLT in the context of SPLTV is needed to fill this gap and improve the effectiveness of mathematics learning.

The purpose of this research is to develop a learning trajectory for the topic of systems of linear equations in three variables, known as a hypothetical learning trajectory. This trajectory is designed to

improve the quality of learning by considering the learning obstacles that arise in the study of Three-Variable Linear Equation Systems, based on the didactical design research approach.

METHODS

This research is a form of Didactical Design that adopts a qualitative method, taking into account the obstacles in the learning process based on previous studies and adjusted to the characteristics of students. According to Suryadi (2019), there are several stages in Didactical Design research, including prospective analysis (didactic situation), metapedadidactic analysis, and retrospective analysis. However, this research only reaches the stage of prospective analysis. The research was conducted at SMK Negeri 1 Padaherang, with the subjects comprising 28 class X students who had studied the Three-Variable Linear Equation Systems material and one mathematics teacher. The primary data collected included test results, interviews, and observations, while secondary data were obtained from previously published studies. Data collection methods included observation, interviews with the mathematics teacher, diagnostic tests, and literature reviews.

FINDINGS

The researchers have conducted the study followed by an analysis of the collected data. Learning obstacles were identified through diagnostic tests, while the obtained data were analyzed to design a hypothetical learning trajectory. With the data gathered, the discussion will be divided into three detailed sections:

Analysis of Diagnostic Tests Used to Identify Learning Obstacles

This diagnostic test was conducted in grade 10, consisting of 28 students, and was carried out individually at SMK Negeri 1 Padaherang. After the test was administered, the researcher identified learning obstacles by assigning the code LO (Learning Obstacle). The results of the diagnostic test analysis are presented in Table 1 below.

Table 1. Diagnostic Test Analysis Results

LO Code	<i>Learning Obstacle</i>	Number of Incidents	%
LO.D1	Students do not fully understand the language (vocabulary) of the questions.	6	21,43%
	Students are unable to identify the important elements in the questions.	12	42,86%

LO.D3	Students do not understand the meaning of the questions.	8	28,56%
LO.D4	Students are confused about what should be represented by variables.	15	53,57%
LO.D5	Students cannot write mathematical models.	13	46,43%
LO.D6	Students cannot operate integers accurately.	16	57,14%
LO.D7	Students cannot perform addition and subtraction operations of algebraic forms accurately.	10	35,71%
LO.D8	Students are not accustomed to checking their obtained answers.	26	92,86%
LO.D9	Students do not know how to check their obtained answers.	26	92,86%

Figure 1 below shows the result of one student's answer to a diagnostic test. The student was confused about what should be assumed with variables and did not understand how to relate variables in SPLTV to the mathematical concepts necessary to solve the problem. Research conducted by Wahab & Sunarti (2022) indicates that similar difficulties are experienced by other students in solving SPLTV problems. Their study found that many students struggle to identify appropriate variables and link them with relevant mathematical concepts to solve such problems. Additionally, Simamora, Saragih, & Susilawati (2022) identified that lack of precision and focus also contribute to students' errors in solving SPLTV problems. Fitriah & Ruli (2022) also discovered that immature mathematical reasoning skills lead students to fail in proposing correct variable hypotheses in SPLTV. These findings indicate the need for a more structured and guided approach to teaching SPLTV concepts so that students can understand and apply variables correctly.

Toko PAMAR INDAH Penyelesaian			
1 BK = 20 k			PH ?
2 PN = 6 k = 33.000	X		
1 PH = 3500 k			
3 BK = 18 k = 31.000	X		PE ?
4 PN = 14 k			
1 PS = 55 k			PM ?
2 PH = 6 k = 11.000	X		
toko matahari ceria			
3 BK = 18 k = 31.000	X		PS ?
2 PN = 9 k			
6 BK = 36 k = 54.000	(6k) X		PB ?
4 PN = 18 k	(18k)		
3 BK = 18 k			PL ?
4 PN = 6 k = 12.000	6k		
3 PH = 18 k	18		
Jadi toko PAMAR INDAH lebih murah daripada			
Pantai ceria karena perbandingan harga X			

Figure 1. The student responses on LO.D4

Furthermore, another obstacle students face is their inability to articulate mathematical models. Students struggle to formulate accurate mathematical equations that represent the relationships between variables in SPLTV. This difficulty is evident in the student responses shown in Figure 2.

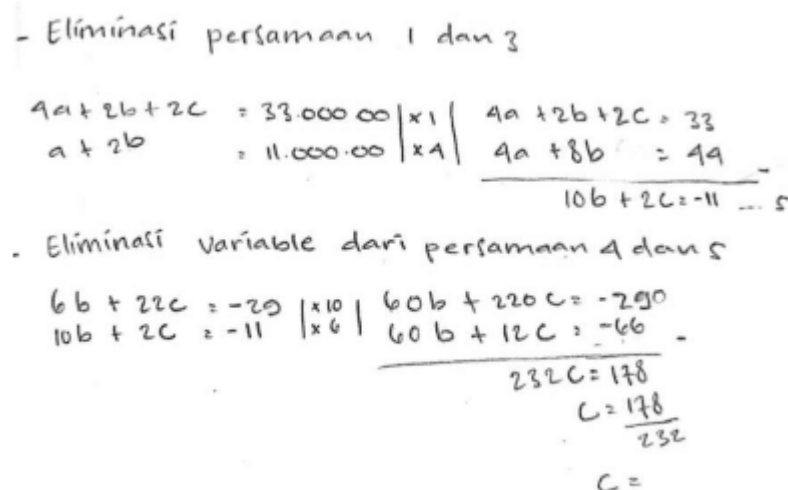
Penyelesaian	
4 = BK = 24 . k	Toko P1
2 = PH = 8 . k	= 33.000 (PH)
2 = PN = 3000 k	
3 = BK = 18 . k	
4 = PN = 14 . k	= 32.000 (PE)
1 = BK = 6 . k	
2 = PH = 5 . k	= 11.000 (PM)

Figure 2. The student responses on LO.D5

Previous research by Bakar, Suryadi, & Darhim (2019) emphasized that students often face challenges in formulating mathematical representations, which are crucial for solving problems involving linear equations. Understanding these difficulties underscores the need for appropriate teaching strategies to enhance students' mathematical modeling skills in SPLTV.

In this context, the development of a Hypothetical Learning Trajectory (HLT) for the Three-Variable Linear Equation System (SPLTV) can be an effective solution. HLT is a step-by-step instructional plan designed to address students' difficulties in understanding and solving complex mathematical problems such as SPLTV (Husna, Zulkardi, Putri, Susanti & Nusantara, 2024). This approach focuses on well-planned learning stages, allowing students to gradually build a deeper understanding of challenging mathematical concepts, such as relationships between variables in SPLTV. Therefore, the implementation of HLT can help improve students' mathematical modeling skills, enabling them to be more confident and competent in handling mathematical problems involving linear equations.

The obstacle faced by students in completing the SPLTV material lies in one of its prerequisites. Students have not yet mastered integer operations accurately. Their ability to perform basic mathematical operations such as addition, subtraction, multiplication, and division with integers still needs improvement. This is evident in Figure 3.



- Eliminasi persamaan 1 dan 3

$$\begin{array}{rcl} 4a + 2b + 2c & = & 33.000.00 \\ a + 2b & = & 11.000.00 \end{array} \quad \begin{array}{l} \times 1 \\ \times 4 \end{array} \quad \begin{array}{rcl} 4a + 2b + 2c & = & 33 \\ 4a + 8b & = & 44 \end{array}$$

$$10b + 2c = -11 \dots 5$$

- Eliminasi variable dari persamaan 4 dan 5

$$\begin{array}{rcl} 6b + 22c & = & -29 \\ 10b + 2c & = & -11 \end{array} \quad \begin{array}{l} \times 10 \\ \times 6 \end{array} \quad \begin{array}{rcl} 60b + 220c & = & -290 \\ 60b + 12c & = & -66 \end{array}$$

$$232c = 178$$

$$c = \frac{178}{232}$$

$$c =$$

Figure 3. The student responses on LO.D6

Previous research indicates that many students struggle with mastering basic mathematical operations, including addition, subtraction, multiplication, and division of integers (Shah, Syarifuddin, Hamzah, & Handayani, 2023). The findings of this study demonstrate that most students still need to enhance their proficiency in these areas, which is crucial for understanding more complex mathematical concepts such as SPLTV. Lack of comprehension in these fundamental operations can impact students' ability to effectively solve mathematical problems involving SPLTV.

Shah, Syarifuddin, Hamzah, & Handayani (2023) assert that students' proficiency in basic mathematical operations like addition, subtraction, multiplication, and division of integers needs further improvement to better grasp the SPLTV concept. This underscores the need for a structured and



sustained learning approach, such as a Hypothetical Learning Trajectory (HLT), specifically designed to address students' difficulties in comprehending complex mathematical topics like SPLTV. HLT can provide well-planned learning steps, allowing students to gradually build a deep understanding of the relationships among variables in SPLTV. Through HLT implementation, it is expected that students can develop better mathematical modeling skills, enabling them to approach and solve mathematical problems with greater confidence and competence.

Interview

In this study, interviews were conducted with one mathematics teacher of SMK Negeri 1 Padoherang, following the interview guidelines previously used in research by Putri (2018:110). After the interview process, the following results were obtained:

1. Students have a limited understanding of the prerequisite material for Three-Variable Linear Equation Systems.
2. Students do not yet grasp the concept of Three-Variable Linear Equation Systems.
3. There are no references available other than the books provided by the school.
4. There is insufficient use of instructional media when teachers explain Three-Variable Linear Equation Systems materials.

The Design of Hypothetical Learning Trajectory (HLT)

Based on the discovery of learning barriers, the researcher proceeded to design a Hypothetical Learning Trajectory (HLT) grounded in these identified challenges. The HLT crafted by the researcher comprises three primary components: learning goals, student activities, and anticipated achievement predictions. Subsequently, the researcher drafted an initial draft related to Three-Variable Linear Equation Systems material, guided by the identification of student learning barriers. Here is an example of the initial HLT draft as detailed in Table 2.

Table 2. HLT based on Learning Obstacle

Learning Objectives	Description of Activities	Students's Thought Conjecture
Understanding the relationship between variables in a system of three-variable linear equations.	<ul style="list-style-type: none">- The teacher begins with an introduction to the concept of variables and their relationship in mathematics with simple examples such as temperature, time, and prices of goods.- Students are tasked with creating a list of keywords or mathematical vocabulary relevant to the concept of three-variable linear equation systems.- The teacher provides a visual presentation using an interactive whiteboard or other	Students may ask questions about how three variables influence each other in an equation.

	media to illustrate the concept of the relationship between variables in three-variable linear equation systems.	
Being able to interpret solutions to a system of three-variable linear equations in real-world contexts.	<ul style="list-style-type: none"> - The teacher provides a series of problems or real-world situations where students have to solve systems of three-variable linear equations. - Students work individually or in small groups to apply the previously learned solution methods to solve these problems. - Upon completion, students are asked to share their solutions and discuss their interpretations of the results in the context of the real world. 	Students start to apply the taught methods and inquire about situations where the method is ineffective.
Enhancing student engagement in the mathematics learning process.	<ul style="list-style-type: none"> - The teacher organizes a group discussion session where students are given mathematical problems involving a system of three-variable linear equations to be solved collaboratively. - Students are instructed to formulate their own solutions and explain their reasoning to other group members. - The teacher facilitates the discussion by providing guiding questions and encouraging students to critically think about various solution approaches. 	Students participate well in group discussions.
Ensuring students have a strong understanding of basic mathematical concepts.	<ul style="list-style-type: none"> - The teacher provides additional exercises focused on understanding the language of mathematics, forming mathematical models, and mastering operations involving integers and algebraic forms. - Afterwards, students are asked to work on these exercises independently. 	Students can complete exercise problems.
Developing students' metacognitive awareness of the mathematics learning process.	<ul style="list-style-type: none"> - The teacher provides exercises in reviewing answers and understanding how to check the obtained answers. Students are asked to work on these exercises independently and to note down the processes they undergo. - Afterwards, students are requested to share their experiences in checking the answers and reviewing the processes they followed. 	Students review their answers afterward.

DISCUSSION

In this study, diagnostic test analysis revealed several learning obstacles faced by students, such as difficulty in comprehending questions thoroughly, identifying key elements in questions, and understanding the meaning of questions. These findings align with Puspita et al.'s research (2023), which highlights students' difficulties in understanding mathematical concepts and solving mathematical problems. Interview results further elucidated learning obstacles, such as a lack of

understanding of prerequisite material and a scarcity of references beyond school textbooks (Wijaya et al., 2019).

Based on these findings, researchers designed a Hypothetical Learning Trajectory (HLT) to enhance student learning. This HLT aims to address learning obstacles by focusing on conceptual understanding, diverse teaching method application, and metacognitive awareness development among students (Zagoto et al., 2022). The HLT design is consistent with previous research emphasizing the importance of problem-solving-oriented learning approaches and concept application in real-world contexts (Prahmana & Kusumah, 2016). Thus, this study provides a significant contribution to improving the effectiveness of mathematics education by addressing students' encountered learning obstacles.

In developing a Hypothetical Learning Trajectory (HLT) for understanding the previously designed three-variable linear equation system, several student learning barriers can be identified and addressed as follows:

1. Students do not fully understand the language (vocabulary) of the problem.

HLT addresses this by starting with the introduction of variable concepts using simple examples, such as temperature or commodity prices, to help students understand relevant mathematical terms. Previous research indicates that students' difficulties in understanding the mathematical language or vocabulary in problems can be addressed with educational approaches such as Hypothetical Learning Trajectory (HLT). HLT initiates learning by introducing variable concepts through simple examples closely related to students' daily experiences, such as temperature or commodity prices. This approach helps students to more easily understand and internalize relevant mathematical terms within the problem context (Sari, Saragih & Napitupulu, 2024). The study suggests that starting from concrete and everyday examples allows students to more quickly connect and apply mathematical concepts more effectively in mathematical problem-solving (Simon, 1995). This aligns with constructivist theory in mathematics education, emphasizing the importance of linking learning to students' direct experiences to enhance their understanding of mathematical concepts.

2. Students have difficulty identifying key elements in problems.

HLT provides tasks to create lists of relevant mathematical keywords, helping students identify essential elements in the context of three-variable linear equation systems. Students often struggle with identifying critical components within complex problem scenarios, particularly evident in contexts involving systems of linear equations with three variables. Research indicates that educational strategies such as Hypothetical Learning Trajectory (HLT) play a pivotal role in addressing these challenges. By structuring learning tasks that prompt students to compile lists of pertinent mathematical terms and concepts related to three-variable systems, HLT fosters a deeper understanding of problem contexts (Hasanah & Rosyidi, 2023). This approach not only enhances

students' ability to discern crucial information but also strengthens their capacity to formulate effective strategies for solving mathematical problems, aligning with constructivist principles that advocate for active engagement and contextualized learning experiences.

3. Students do not understand the purpose of problems.

Teachers use visual presentations to illustrate the concept of variable relationships in real mathematical situations, helping students understand the context and purpose of each problem. Recent studies emphasize the efficacy of visual representations in elucidating the intricate relationships between variables within mathematical contexts. By employing visual aids such as diagrams, graphs, and charts, educators can effectively demonstrate how variables interact and influence outcomes in real-world scenarios. This approach not only enhances students' comprehension of the underlying concepts but also clarifies the relevance and application of mathematical problems. Research supports that visual representations foster a deeper understanding of variable relationships, empowering students to discern the purpose of each problem by connecting abstract mathematical concepts to tangible, contextual situations (Parame-Decin, 2023). Integrating visual presentations in teaching practices cultivates a holistic learning environment where students grasp the significance of variables in problem-solving, thereby strengthening their overall mathematical proficiency.

4. Students are confused about what variables should represent.

Through exercises in HLT, students gain direct experience in solving mathematical problems involving variables, facilitating their understanding of the role of variables in mathematics. Through exercises in HLT (Hypothetical Learning Trajectory), students engage in structured activities that demystify the concept of variables by contextualizing them within familiar scenarios. Research highlights that HLT methodologies guide students in identifying and defining variables based on problem context, thus reducing confusion about their roles in mathematical expressions and equations. By practicing with concrete examples such as real-world applications of algebraic concepts, learners develop a clearer understanding of how variables represent unknown quantities and parameters in mathematical models. This approach not only enhances comprehension but also nurtures students' ability to apply algebraic principles across diverse problem-solving situations, reinforcing their mathematical proficiency.

5. Students cannot write mathematical models.

Teachers provide additional exercises in HLT focusing on developing the skill to accurately write mathematical models. Recent studies have emphasized the role of Hypothetical Learning Trajectory (HLT) in addressing students' challenges in writing mathematical models. HLT frameworks offer structured sequences of learning activities designed to enhance students' proficiency in formulating accurate mathematical models (Apriyanti, Suweken, & Suparta, 2019). These exercises are crucial



as they provide learners with repeated opportunities to practice translating real-world scenarios into mathematical representations, thereby fostering clarity and precision in model construction. By guiding students through progressively complex tasks within a supportive learning environment, teachers enable them to develop the necessary skills to articulate mathematical concepts effectively. This approach not only supports deeper comprehension of mathematical principles but also cultivates students' ability to apply these models across various contexts, reinforcing their overall mathematical competence.

CONCLUSION AND SUGGESTION

Based on the previous information, it can be concluded that students experience learning difficulties in the Three-Variable Linear Equation Systems material, influenced by various factors such as limited learning resources, delivery of material not aligned with the curriculum, and students' lack of understanding of prerequisite material. In response to these research findings, a learning trajectory or Hypothetical Learning Trajectory (HLT) has been developed for the Three-Variable Linear Equation Systems material, structured based on the identification of learning barriers to help address the difficulties faced by students. The HLT developed in this study includes 5 components, including understanding the relationship between Three-Variable Linear Equation Systems variables, interpreting Three-Variable Linear Equation Systems solutions, increasing student engagement in the learning process, students' understanding of basic mathematical concepts, and developing students' metacognitive awareness of the mathematics learning process.

REFERENCE

- Apriyanti, M., Suweken, G., & Suparta, I. N. (2019). Development Of Hypothetical Learning Trajectory For Linear Equations With Pisa And Scientific Approach Model Consideration. *Jurnal Pengajaran MIPA*, 24(2), 78-86.
- Asfya, I. B. Desain Pembelajaran Sistem Persamaan Linier Tiga Variabel Dengan Konteks Membuat Hiasan Hijab Berbantuan Ict Untuk Siswa Smk.
- Bakar, M. T., Suryadi, D., & Darhim, D. (2019). Learning obstacles on linear equations concept in junior high school students: Analysis of intellectual need of DNR-based instructions. In *Journal of Physics: Conference Series* (Vol. 1157, No. 3, p. 032104). IOP Publishing.
- Fitriah, Z., & Ruli, R. M. (2022). Kemampuan Penalaran Matematis Siswa Dalam Menyelesaikan Permasalahan Hots Pada Materi Sistem Persamaan Linear Tiga Variabel. *JPMI (Jurnal Pembelajaran Matematika Inovatif)*, 5(3), 915-928.
- Hasanah, K., & Rosyidi, A. H. (2023). The Process of System of Linear Equations in Three Variables Solving Procedure's Construction Using Analogy: Individual VS Paired. *MATHEdunesa*, 12(2),



534-556.

- Hasibuan, I. (2015). Hasil Belajar Siswa Pada Materi Bentuk Aljabar Di Kelas Vii Smp Negeri 1 Banda Aceh Tahun Pelajaran 2013/2014. *Jurnal Peluang*, 4(1).
- Hidayati, I., Deciku, B., & Azizah, T. (2022). Hypothetical Learning Trajectory Sistem Persamaan Linear Dua Variabel Berbasis Realistic Mathematics Education. *Juring (Journal For Research In Mathematics Learning)*, 5(2), 109-118.
- Husna, A., Zulkardi, Z., Putri, R. I. I., Susanti, E., & Nusantara, D. S. (2024). Hypothetical learning trajectory of linear equation systems with three variables: The context of typical snacks riau islands. *Jurnal Math Educator Nusantara: Wahana Publikasi Karya Tulis Ilmiah di Bidang Pendidikan Matematika*, 10(1), 12-22.
- Kusumawati, E., & Aulia, F. (2018). Identifikasi Kesulitan Menyelesaikan Soal Fungsi Komposisi Peserta Didik Kelas X Keuangan Smk Negeri Di Banjarmasin Tahun Pelajaran 2016/2017. *Edu-Mat: Jurnal Pendidikan Matematika*, 5(2).
- Lantakay, C. N., Senid, P. P., Blegur, I. K. S., & Samo, D. D. (2023). Hypothetical Learning Trajectory: Bagaimana Perannya Dalam Pembelajaran Matematika Di Sekolah Dasar?. *Griya Journal Of Mathematics Education And Application*, 3(2), 384-393.
- Maarif, S., Setiarini, R. N., & Nurafni, N. (2020). Hambatan epistemologis siswa dalam menyelesaikan masalah sistem persamaan linear dua variabel. *Jurnal Didaktik Matematika*, 7(1), 72-89.
- Muthmainnah, M. (2023). Analysis of Students' Mastery of Mathematical Concepts Material for Linear Equations in Two Variables. *Tarbiyah: Jurnal Ilmiah Kependidikan*, 12(2), 135-147.
- Nurhasanah, H., Prabawanto, S., & Sumiaty, E. (2019). Didactical design development of two variables linear equation. (*JIML*) *Journal Of Innovative Mathematics Learning*, 2(4), 186-193.
- Parame-Decin, M. B. (2023). Visual Representations in Teaching Mathematics.
- Prahmana, R. C. I., & Kusumah, Y. S. (2016). The Hypothetical Learning Trajectory On Research In Mathematics Education Using Research-Based Learning. *Pedagogika/Pedagogy*, 123(3), 42-54.
- Pratamawati, A. (2020). Desain didaktis untuk mengatasi learning obstacle siswa sekolah menengah atas pada materi fungsi invers. *Jurnal Pendidikan Matematika (Kudus)*, 3(1), 15-28.
- Puspita, E., Suryadi, D., & Rosjanuardi, R. (2023). Learning Obstacles Of Prospective Mathematics Teachers: A Case Study On The Topic Of Implicit Derivatives. *Kreano, Jurnal Matematika Kreatif-Inovatif*, 14(1), 174-189.
- Putri, I., S.(2018). Desain Didaktis Pembelajaran Matematika Untuk Mengatasi Hambatan Epistemologis Pada Konsep Program Linear Di Sma (Bachelor's Thesis, Jakarta: Fakultas Ilmu Tarbiyah Dan Keguruan Uin Syarif Hidayatullah).
- Rahayu, T., & Fuadiah, N. (2021). Hypothetical Learning Trajectory Pada Pembelajaran Peluang Smp Menggunakan Konteks Permainan Tepuk Gambar. *Jurnal Didaktis Indonesia*, 1(2), 93-103.



- Risqullah, F., & Hidayah, N. (2024). Analysis Of Student's Worksheet Needs On Material Problem Systems Of Linear Equations With Two Variabel For Mathematics Literacy Skills. In *Proceeding International Conference on Islam and Education (ICONIE)* (Vol. 3, No. 1, pp. 893-902).
- Sari, N. (2023). Exploration of The Didactic Situation of Linear Equations in SMP Suci Murni Medan. *Mathline: Jurnal Matematika dan Pendidikan Matematika*, 8(2), 569-588.
- Sari, N., Saragih, S., & Napitupulu, E. E. (2024). Developing a Hypothetical Learning Trajectory with Problem-Based Learning and a Learning Medium for Middle School. *Educational Administration: Theory and Practice*, 30(1).
- Septiana, R., Kesumawati, N., & Fuadiah, N. F. (2021). Hypothetical Learning Trajectory Berorientasi Asesmen Kompetensi Minimum Pada Pembelajaran Sistem Persamaan Linear Tiga Variabel. *Jurnal Didaktis Indonesia*, 1(2), 83-92.
- Simamora, Y., Saragih, R. M. B., & Susilawati, S. (2022). Analisis Kesalahan Siswa Menyelesaikan Soal Tes Kemampuan Pemecahan Masalah Matematis Kelas X SMK Swasta Muhammad Yaasiin Sei Lapan. *Omega: Jurnal Keilmuan Pendidikan Matematika*, 1(1), 1-6.
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for research in mathematics education*, 26(2), 114-145.
- Suryadi, D. (2016). Monograf Didactical Design Research. Bandung: Rizqi Press.
- Suryadi, D. (2019). Monograf 2 Didactical Design Research (Ddr). Bandung: Gapura Press.
- Shah, K., Syarifuddin, A., Hamzah, A., & Handayani, T. (2023). Analisis Kesulitan Belajar Matematika Materi Operasi Bilangan Bulat Pada Siswa Sekolah Dasar. *JUDIKDAS: Jurnal Ilmu Pendidikan Dasar Indonesia*, 2(3), 161-170.
- Wahab, A., & Sunarti, S. (2022). Diagnosa kesulitan siswa dalam menyelesaikan soal spltv melalui pemetaan kognitif berbasis polya dan upaya mengatasi dengan scaffolding. *RAINSTEK: Jurnal Terapan Sains & Teknologi*, 4(1), 1-14.
- Wahyuni, D., & Tasman, F. Pengembangan Modul Pembelajaran Matematika Berbasis Pendekatan Realistic Mathematics Education (Rme) Materi Barisan Untuk Meningkatkan Kemampuan Pemahaman Konsep Peserta Didik Kelas Xi Sma.
- Wicaksono, A., Prabawanto, S., & Suryadi, D. (2024). How students' obstacle in solving mathematical tasks deal with linear equation in one variabel. *Al-Jabar: Jurnal Pendidikan Matematika*, 15(1), 33-44.
- Wijaya, A., Retnawati, H., Setyaningrum, W., & Aoyama, K. (2019). Diagnosing Students' Learning Difficulties In The Eyes Of Indonesian Mathematics Teachers. *Journal On Mathematics Education*, 10(3), 357-364.
- Zagoto, M. M., Arnawa, I. M., Musdi, E., Edi, F., Dakhi, O., & Verawardina, U. (2022). Hypothetical Learning Trajectory Berbasis Realistics Mathematics Education. *Edumaspul: Jurnal*



Pendidikan, 6(2), 3098-3104.