IJEIR
International Journal of Educational Innovation and Research
Volume 3, Number 1, 2024, pp. 27-40
DOI: https://doi.org/10.31949/ijeir.v3i1.6905
ISSN: 2810-0808

Students' Inconsistency when Solving a Geometry Problem in Three-Dimensional Context

Fiki Alghadari[1]*, Abdul Hakim Ma'ruf[1], Audi Yundayani[2]
[1]Mathematics Education Study Program, STKIP Kusuma Negara, Jakarta, Indonesia
[2]English Education Study Program, STKIP Kusuma Negara, Jakarta, Indonesia
*Corresponding Author: fiki_alghadari@stkipkusumanegara.ac.id

ABSTRACT
There is an issue regarding students' consistency in completing geometry assignments, as indicated by several research findings and assessments. Diagnosing how students integrate concepts in the conceptual design and understanding the reasons behind their inconsistency in completing assignments are the main focuses of this case study. This research was conducted with 58 high school students in Tanjungpandan, Indonesia. The data were obtained from students' answers to problems of the three initial levels of geometry thinking and retrospective reports about their answers. The data were analyzed based on three phases: the concept-eliciting and integrating phase, the relationship-eliciting phase, and the relationship-integrating phase. The study revealed that students' performance in geometry analysis aligned with the epistemological concept issue. Visual objects garnered the most attention from students, leading to their analysis techniques being primarily object-oriented. Some stages of property analysis were skipped, causing students to make claims about objects of thought when they should have been establishing relationships between properties to classify shapes through rigorous geometry analysis. Numerical computation remains an essential aspect of geometry analysis. The conceptual design has not yet reached the abstraction stage, resulting in experiments to solve problems not always yielding the correct solutions. In education, this highlights the need for a deep understanding of concept epistemology, efficient concept integration, and the cultivation of abstract thinking skills.

ARTICLE HISTORY
Received 2023-10-02
Accepted 2023-12-15

KEYWORDS
Conceptual design
Geometry education
Problem solving
Three-dimensional

INTRODUCTION

Geometry in school studies topics such as points, lines, and planes, including their potential extension into three-dimensional space. Geometry has a fundamental function, which is to develop competencies in relating language, thought and abilities in building up rigorous reasoning (Mammana et al., 2012). In the curriculum of Indonesian senior high school mathematics education, geometry is one of the content elements that addresses the shapes of plane figures and three-dimensional structures, both in Euclidean and Non-Euclidean studies, as well as their characteristics in the sub-elements of plane geometry and spatial geometry (Kemdikbudristek, 2022). In the study of three-dimensional geometry, students are expected to master the material on the distance between two points, points and lines, as well as points and planes. This

This is an open access article under the CC BY-SA license.
Copyright © 2024 by Author.

ability plays a crucial role in laying the foundation for learning vectors (F. Alghadari & Herman, 2018), or developing skills in the field of science (Romenskyy et al., 2020; Sahaf et al., 2021; Wu et al., 2023).

In the context of learning three-dimensional geometry in high school, some research findings state that: (1) the instructional media using PowerPoint accompanied by developed Visual Basic for Applications has been deemed valid and practical, demonstrating a potential impact on students' cognitive abilities in three-dimensional geometry (Marfuah et al., 2016; Retta & Fitriasari, 2022); (2) the learning model with the syntax of engagement, exploration, explanation, elaboration, and evaluation with a scientific approach shows better students' achievement in learning geometry compared to problem-based learning or direct learning (Paryatun et al., 2016); (3) most of the students' difficulties arise from insufficient attention to notation and issues of calculation accuracy, that after the implementation of the think-pair-share learning approach, so students' mathematical communication reaches proficiency, improves, and is significantly better compared to the expository learning group (Haryanti, 2018); (4) The use of the GeoGebra application has significantly enhanced or influenced the learning outcomes in three-dimensional geometry (Nur et al., 2023; Yuliani et al., 2021). All of that is based on data and have been convinced on the basis of their respective analyzes result that geometry achievement of students in secondary schools is better.

On the other hand, the low achievement of geometry in schools has become a critical issue in mathematics education research in the world (Hock et al., 2015). In Indonesia, the mathematics achievement of 15-year-old students in the 2018 PISA survey is at a score of 379, below the OECD average (489), only 28% of students in Indonesia at least they can interpret and recognise, without direct instructions, how a (simple) situation can be represented mathematically, and students have shown relatively stable performance in mathematics since 2009 (OECD, 2019). One of the other reports, the results of the 2019 national examination assessment by the educational assessment center (Pusat Penilaian Pendidikan, Puspendik, 2019) that the percentage of students of science and social studies programs can correctly answer geometry questions, each reaches 34.59% and 21.82%, of the total number of students in each program. According to the report, it was noted that: (a) science program students who were able to correctly answer the geometry problem with indicators determining the distance of points to lines in the cube were 30.85%, (b) science program students who were able to determine the distance of points and plane in the cube were 49.50%, (c) students of social programs who can answer correctly about geometry with an indicator showing the distance between points to the plane of a building is 29.56%, and (d) students of social programs can determine the length of diagonal on solid is 33.87%. Meanwhile, according to the national assessment data in 2022, It is known that the percentage of secondary school students achieving the minimum numeracy competency is 40.63% at the junior high school level, showing an increase of 3.79%, and 41.14% at the senior high school level, with an improvement of 5.98% from the previous year (Kemdikbud, 2023).

The achievement of Indonesian high school students in geometry, according to survey results and several research studies, has been found to be not compatible with each other. This indicates the relevance and inconsistency of students in completing geometry task. Because of the problem of consistency, there is a certain part of relevance between students' concept knowledge, abilities, and learning processes that should be considered for their cognitive activities in class geometry (Fiki Alghadari et al., 2020; Isnawan et al., 2022; Noor & Alghadari, 2021a), including representation of 3D shapes, spatial structuring, conceptualization of mathematical properties, and measurement (Pittalis & Christou, 2010). Therefore, some researchers have recommended the need for an analysis of how students conceptualize problem solving to explain the phenomena of cognitive restructuring and conceptual reorganization when they engage in complex domains that describe problem solving behavior and affect their performance (Biccard, 2018; Suwa, Gero, & Purcell, 1998; Yee, 2017).

The task of diagnosing learning problems in students is fundamental and crucial to improve their learning achievements (Hwang et al., 2012; Nurhikmayati et al., 2022). Various studies have been conducted, such as Isnawan et al. (2022), who explored the inconsistency in the meaning of fractions and the use of fractional representation models due to limitations in understanding, thus hindering students' opportunities to comprehend other meanings. The study by Nurhikmayati et al. (2022) on learning obstacles found that the definition of the concept of geometric transformation is inconsistently interpreted due to students' understanding of the concept or the presence of cognitive conflicts. Lin et al. (2004) investigated inconsistencies about students' reasoning, proving, and understanding proof in number patterns, indicating that students need to develop skills to connect heuristic ideas and procedural ideas to formal proofs by adhering to logical steps and correct mathematical procedures. However, based on the available literature to date, we could not find research that primarily focuses on students' inconsistencies in solving geometry problems, particularly in three-dimensional figures.

Solving problems is conducted with new knowledge by constructing representations that bridge the initial and goal states (Ward, 2012). Solving a problem is the same mean to schematizing concepts and component of information from the initial and goal states. Information on initial and goal states is conceptualized so that binding and connecting bonds are characterized by the emergence of conceptual tools as an appropriate solution (Biccard, 2018; Lesh & Harel, 2003). The solution is the goal of conceptual design in solving problems (Alghadari et al., 2020; Dossey, 2017). Guarino, Oberle, and Staab (2004) states that the conceptualization process is about what is conceptualized for a purpose. Thus, solving problems is a way to conducted new knowledge which is constructed through the conceptualization process (Dossey, 2017; Fauskanger & Bjuland, 2018; Simon, 2017; Ward, 2012). When students have problems with the conceptualization process, problem solving abilities become inconsistent due to the complexity of cognitive ability to process conceptual knowledge (Noor & Alghadari, 2021b; Yee, 2017). That can happen because there is a basic knowledge whose construction is not complete, or the conceptual knowledge is complete but there is a misconception (Rahayu & Alghadari, 2019; Rosilawati & Alghadari, 2018). As a result, students are disrupted in solving problems, advanced knowledge is difficult to be developed through problem solving activities (Veloo, Krishnasamy & wan Abdullah, 2015). Therefore, this research was conducted to study the process of solving students' geometry problems. How the conceptual design they do (research question, RQ1), how the concept is integrated for completion (RQ2), and why students are inconsistent in solving geometry problems (RQ3), are some of the study questions.

METHODS

This research was conducted using a case study perspective based on Edmonds and Kennedy's (2016) model, which investigates students' conceptualization processes in solving problems for various geometry tasks. The studied cases are in: (1) conceptual design, (2) concept integration, and (3) inconsistency modeling, through three test items that will be completed by the students. Three items of geometry problems were prepared based on the three initial levels of van Hiele's thinking namely visualization, analysis, and abstraction. Students complete the test with pen and paper. They also write down their reasons for retrospective reports about the answers made as a form of epistemological clarification, which as a benchmark of a description of the conceptualization process that they do.

Students' Participants

There were a total of 58 social program students at Tanjungpandan Public High School involved in this study. These students gave their responses to geometry problem. The data we need pertains to students' responses to problems that depict the problem solving process and are related to consistency issues in the solution process. Therefore, the focus of our attention is on the source of the problem for students in the process of solving that they consistently do in conceptualization. We analyzed a set of data, paying attention to reduce redundancy due to the similarities in the problem solving processes students employed (the similar model answer), and ultimately identified three students whose responses generally met the criteria. Three students come from a rural area very far from the center of the capital city so that the results of the study can also illustrate the problem of equal distribution of student ability levels based on secondary level education units. Their names are initials with AD, AL, and GD. They are the final level students in preparation for the final stage of Education exams that will be carried out using a computer-based test but the process of completing the task is not software-assisted. They have learned geometry in building space. They are all male students and do not conflict with the results of the study that the achievements of male students are more than female (Alghadari & Herman, 2018). We use a sample from one of the sex categories because aside from rarely encountering spatial issues in solving problems that involve three-dimensional models such as cubes or pyramids, their response to the problem also meets the required data criteria.

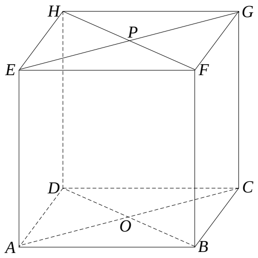
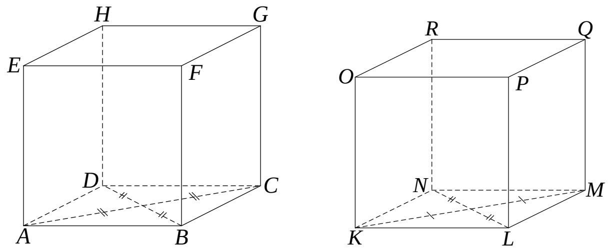
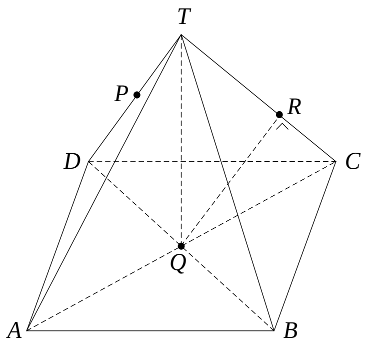
Instrument Specification

In the high school mathematics education curriculum, students learn three-dimensional material about the distance between: two points, points and lines, as well as points and the plane. They have accepted the definition of the distance between two geometry elements to note that it is the closest distance between the two elements in question. The instrument used is the problem of geometry in three-dimensional shapes which are prevalent as models in assignments and in final student assessment exam questions. In this study, we employed three geometry problems within the context of three-dimensional figures, each representing one of the three levels of van Hiele. (van de Wall dkk., 2017; Hock dkk., 2015; Herbst dkk., 2017; Alghadari, Herman & Prabawanto, 2020), namely visualization, analysis, and abstraction, were specified with objects and products of thought, aligning with the levels of geometry task indicators in Puspendik (2019). The specifications of the three problems are as follows.

At the visualization level, since the question pertains to the distance between the line *AE* and the plane *BDHF*, the objects of thought are the geometric planes within the mental representation of the cube *ABCD.EFGH*. Based on the definition of the distance between two geometric elements, in reasoning to answer this question, students have the opportunity to consider other objects such as: the plane *ACGE* because it includes *AE*, *AO*, and *EP*; the plane *ABCD* because it includes *AO*; or the plane *EFGH* because it includes *EF*. Based on these planes and referring to Van de Walle et al. (2017) that the products of thought at this level were classified as a class of shapes, so that some products of thought are the planes *BDHF* and *ACGE* as the rectangles, while *ABCD* and *EFGH* as the squares.

At the analysis level, the objects of thought are squares and rhombi, each displayed on the mental representation screen of the cube or prism. The quadrilateral planes constitute a class of shapes. There are numerous line segments on the cube and prism with positions parallel to others that are contained in squares or rhombi. The students have the opportunity to analyze various comparisons to identify the differences between the two figures. For example, the line segment *AC* in square *ABCD* is parallel to the line segment *EG* in square *EFGH*, or the line segment *AC* is parallel to the line segment *EG* in rectangle *ACGE*, and their distance is *AE* or *CG* in the cube *ABCD.EFGH*, compared to the line segment *KM* in rhombus KLMN parallel to the line segment *OQ* in rhombus *OPQR*, or *KM* parallel to *OQ* in rectangle *KMQO*, and their distance is KO or MQ in the prism. In the figures, it is known that *AE* and *CG* are of the same length as *KO* and *MQ*, but *AC* and *GE* are not of the same length as *KM* or *OQ*. In other words, the distances between the parallels are the same, but their lengths are different. This difference in length is a property of shape between the diagonals in a square and the diagonals in a rhombus. According to Van de Walle et al. (2017), this property of shape is a product of thought.

At the abstraction level, the objects of thought are properties of shape, while the products of thought are relationships between those properties (Van de Walle et al., 2017). For example, some properties of shape that become objects of thought are *QR* perpendicular to *TC* in triangle *TQC*, P in the middle of *TD*, and in the middle of *BD* in triangle *TBD*. Based on these properties of shape, the product of thought in triangle *TQC* is the comparison of the area of the triangle between *QC* as the base with *TQ* as the height and TC as the base with *QR* as the height. Meanwhile, the product of thought in triangle *TBD* is the comparison of the lengths *DP* and *PQ* equal to *DT* and *TD*. Furthermore, the distance from P to *QR* is the relationship in triangle PQR with its height, and it is also a product of thought.

Category	Questions
Visualization	Observe the cube *ABCD EFGH*.  What line segment represents the distance between line *AE* and the plane *BDHF*? Why?
Analysis	Figure (a) is the cube *ABCD EFGH*. Figure (b) is the prism with a rhombus base *KLMN OPQR*. The length of all edges in both Figure is the same.  (a) (b) How do the two Figure above differ in terms of the distance between two parallel lines? Why?
Abstraction	Given that a square pyramid *T.ABCD* with equal sides *AB*=*BC*=*CD*=*DA*=4cm. The lengths of all the slant edges of the pyramid are 2√6cm. *Q* is the intersection of *AC* and *BD*, *P* is in the middle of *TD*, and *R* is on *TC* such that *QR* is perpendicular to *TC*. What is the distance between *P* and *QR*? 

Data Analysis

Firstly, identify data based on the three phases of the learning problem diagnosis model suggested by Hwang et al. (2012), that is: (1) the concept of eliciting and integrating phase [Type A], (2) the relationship-eliciting phase [Type B], and (3) the relationship-integrating phase [Type C]. The process of designing conceptual of completion by students was diagnosed based on these three phases for each geometric thinking level, resulting in the classification of students in each respective phase. Secondly, systematizing the problem solving process and investigating the reasons in the retrospective report from students classified as Type A, Type B, and Type C, thus selecting student responses that represent making conclusions regarding the research questions. Thirdly, identifying differences in conceptual designs based on the systematics of completion for consideration of the variations in integrated concepts. Guarino et al. (2004) state that conceptualization is expressing the shared views of several parties and consensus rather than individual views. Quoting the statement, this study is analyzed using constant comparative techniques, which are comparing processes in the cases of students. Critical comparisons are made on the writing of students themselves and others to find relationships between processes and patterns of concept integration. Analysis of integration patterns not only on given concepts but also relevant concepts for applying to solve. The case findings on integration patterns are part of the problem of consistency. Fourth, mapping inconsistency points.

RESULTS AND DISCUSSION

The variety of student responses to geometry problems, the classification results based on the phases of Hwang et al. (2012) are: (a) there were problems with the concept eliciting and integrating phase [Type A], (b) there were problems with the relationship-eliciting phase [Type B], and (c) there were problems with the relationship-integrating phase [Type C], presented in Table 1.

Table 1. Classification of student responses based on type

Type	Visualization	Analysis	Abstraction
A	-	26	31
B	-	3	8
C	-	3	4

Table 1 shows the number of student responses encountering difficulties in geometry problems at the analysis and abstraction levels, with the highest number at the abstraction level (43 dari 58 orang). The classification results did not find problem types at the visualization level. This analysis also indicates that the higher the hierarchy level, the more potential there is to increase the number of students experiencing problem types. These findings also confirm the existence of hierarchical levels in the van Hiele theory. The five hierarchical levels of geometric thinking according to the van Hiele theory are visualization, analysis, abstraction, deduction, and rigor (Hock et al., 2015; Herbst et al., 2017; Alghadari, Herman & Prabawanto, 2020). Furthermore, the higher the hierarchy level, the more potential there is to identify inconsistencies in students' approaches to geometry problems.

Next, several subsections are grouped and the description begins about how students respond to the three geometry problems. Based on these responses, illustrated about the conceptualization process of each geometry problem (about RQ1), there is information and concepts at the initial and goal state in the integration of conceptual knowledge, and structuring information schemes will lead students to find concepts that can be integrated to solve problems (about RQ2). A geometry problem solved with an inaccurate process and completion, then the potential is the inconsistency of students solving problems as in the process and then in the process of solving problems that involve the process of thinking geometry at the next level (about RQ3).

Description of Students' Responses to Geometry Problems

At the level of the visualization problem, all students do not show any obstacles they experience in their completion. They can correctly answer the problem at this level. Two students, AL and GD, answered that line *AO* represented the distance between line *AE* and plane *BDHF*. A student, AD, answers differently in that point *R* represents line *AE* and point *Q* represents plane *BDHF*, so the distance between *AE* and *BDHF* is *RQ*. They all reasoned that the distance represented according to each answer was the shortest line segment because *AO* or *RQ* (AD's perspective) was perpendicular to *AE* and *BDHF*.

At the analysis level of geometry thingking problem, one student, GD, did not provide an answer to the analysis level geometry problem [Type A]. AL responds that *AC=BD* but *KM≠LN*. There is another response written by AL, namely: the angles on the side of the cube are always right angles while in the prism it is not so, if for example *O* is the intersection point of *AC* and *BD* and *KM* and *LN* then the isosceles triangle *AOB* is while *KOL* is arbitrary. It is true that it is an isosceles triangle and is arbitrary but AL does not seem to realize that both triangles are right triangles [Type C]. While the response from AD is *AC≠KM*. Another response written by AD is about the difference in the diagonal length of the space and the angle of the two shapes [Type B].

In the abstraction level geometry problem, the initial state in the above geometry problem has been given in the form of information about pyramid *T.ABCD*. Whereas the goal state is the distance between point *P* and line *QR*. The problem is measures such as *QR*, *PQ*, and *PR*. The problem formation is the one based on the three. In the initial stages, all students have started the same steps. They start by paying attention to triangle *QTC*. Triangle *QTC* is one of the geometry transformations for the object of analysis. They know all the lengths of the three sides of the triangle and find relationships between properties so that they can determine the distance between *Q* and *R* with numerical calculations. In the second stage, they focus their gaze to find the distance between *P* and *Q*. This second stage begins to separate their steps.

GD plans to solve this geometry problem involving triangles of *QTC*, *TPQ*, and *QPR*. Through triangle *QTC*, GD precisely determines the length of *QR*. Then GD switches to triangle *TPQ* to calculate *PQ*. GD realizes that *TP* is half of *TD* but the information he uses is wrong because the size of *TD* is not equal to $2\sqrt{6}$ but 4 [Type A]. The next mistake is that GD claims that triangle *TPQ* is a right triangle at *P* so he applies the Pythagorean theorem to calculate *PQ*. Then GD looked at triangle *QPR*, but at the triangle that the length of is not yet known *PR*, there is no continuation about the calculation of *PR* so that the completion process is finished.

AL plans to solve the problem involving triangle *QTC* and triangle *POQ*. In the triangle *POQ* where *PO* is perpendicular to *QR*, where *PO* is perpendicular to *QR* and claims that *QO* is half *QR* then AL claims that *PQ=QR* [Type C]. With several claims by AL so he can apply the Pythagorean theorem to calculate the distance between *P* and *QR*. There are not many triangles involved by AL in the distance calculation plan but there are many claims made so that in the process of completion there are some errors.

AD plans to solve this problem of geometry abstraction by involving four triangles, namely: triangle *QTC* with a right angle at *Q*, triangle *TQD* with a right angle at *Q*, triangle *RPT* with an unwarranted claim that the right angle at *P*, and triangle *QPR* the claimed as an isosceles triangle where *PQ=QR*. After finding the distance between *R* and *Q* through triangle *QTC*, AD plans to calculate *PQ* by paying attention to triangle *TQD*. Although it is not wrong with the length of *QD*, an error occurs when AD states *TD* and *TQ* [Type B], so that it looks the same between triangles *TCQ* and *TQD*. Without doing the calculations, AD concludes that *QP* is equal to *QR* and isosceles triangle *QPR* is. There is an initial position of the error. In addition, claims on triangle *RPT* and errors on the length of *RT* and *PR* so that the problem solving arranged by AD is not in the right calculation path. Because there are errors so the next analysis certainly brings these errors and it can be stated that the geometry problem solving by AD is not right.

Conceptual Design in Problem Solving

Cases at the visualization level, two students design a solution of objects, concepts, and entities by determining the line segment that intersects the edge and a diagonal plane of the cube. After that interpret that the line segment is perpendicular to the two elements of the cube. The conceptualization technique is an object-oriented approach. While one other student drew up the solution, state it as the second conceptualization technique, by defining two notations and the position of the points, each one point on the edge and one point on the diagonal plane of the cube. Next, unite the two points with the line segments in the visual model of the cube with additional symbols to indicate that the line segments are perpendicular to the edge of the cube and diagonal plane. The second conceptualization technique bases its conceptual design with a sketch approach. The goal state of each student on this level of visualization problem may be the same but in the findings of this study that there are differences in the techniques of the three students in the conceptualization design. Line segments that represent the distance between lines and their specified plane with points whose notations are not the same. The dot notation is different but determined from the same visual object, it will result in the emergence of different solutions (Herbst et al., 2017). Different solutions do not always make the wrong solution when all concepts of the core and goal states are integrated in the right way (Van de Walle et al., 2017).

According to students' responses to geometry problems at the analysis level, their conceptual technique is to compare geometric elements in the plane forming visual objects. This is a technique with an object-oriented approach. Objects in student orientation are planes on the sides of the cube and prisms. The area that is the focus of their attention is the second base to build the space. They think about the difference in shape due to the geometry elements of the two visual objects. Al-shehri, Al-Zoubi, and Rahman (2011) and Herbst et al. (2017) stated that geometry thinking starts by paying attention to shapes, followed by analyzing the properties of properties, then understanding the relationships between different shapes, and finally reaching conclusions. The geometry elements in the base plane of the cube and the prism are the source of object differentiation in the conceptual problem of geometry at this level of analysis. In this case, students do not show their greater attention to the concept of the position of parallel lines which should always be involved in their conceptualization design. Even though the parallel lines loaded by the cube and the prism are geometric elements, which conceptually should start focusing the attention of students, to design solutions before they compare the differences or similarities of representation of the concept of the problem. Students pay more attention to the geometry elements on the base.

Cases at the abstraction level, there are four stages to be processed generally. The conceptual design formation is the first 3 stages parallel to one final stage. Xin (2008) calls it a problem scheme which is about a group of problems that share a common underlying structurer requiring a similar solution. In other words, the final stage will be able to be implemented by students after they have succeeded in the three previous stages. Each stage, students will analyze the geometric shapes of different types. Simon (2017) states that different concepts can be determined from the same mathematical relationship. All three students have begun their completion through the same initial stages. But there are differences in design and conceptualization techniques at a later stage. Each student directs the design based on the analysis of different geometric objects but ultimately converges on the same object. The convergence of investigative activities is about where to start and how to proceed. Different geometric shapes have been the focus of attention of each student in his investigation. The focus of student analysis was on triangles. Yilmaz and Argun (2017) state that abstraction is a construction process that in mind involves the provision of relationships between geometric objects and turns these relationships into specific, independent expressions. Based on the triangular shapes at the focus of student analysis, the conceptual design is made by each student in several stages of analysis. The shapes analyzed by the three students were right triangles and scalene triangles. The involved concepts refer to objects of thought, such as the properties of a triangle with its altitude lines In this case, students are thinking to simultaneously obtain a representation of distance and the perpendicularity entity, namely through the altitude of the triangle with its properties, and then perform numerical calculations.

Unfortunately, the organization of concepts in the schema structure by the students has not yielded the correct solution. In the process, students have made baseless and unjustified claims where it is different from conception. conception is to connect the bond between concepts from initial to goal state (Dossey, 2017; Simon, 2017). Simon (2017) states that designing a solution is a conception, both of a concept and between various concepts. With the case, the identity of the concept understood by students shows how they conceptualize (Yilmaz & Argun, 2017). Dossey (2017) and Nikitina (2006) stated that conceptualization is an integrative strategy designed to bring scientific and mathematical thinking beyond a single fact and theory to the level of the underlying concept. Furthermore, Rahayu and Alghadari (2019) stated that it also simultaneously explained his understanding of the concept along with his thinking ability. In the process of solving problems, students conception to connect ties between concepts from the initial to the goal state.

Concept Integration for Solution

At the visualization level, the integrated concepts for the solution emerge in two cases. The emergence of an entity also corresponds to how students integrate it. Two cases of the emergence of integrated concepts and techniques are the findings of this study. In the first case, the concept of rigor is present as a reason after the solution is determined. In this case, there is no guarantee that students know why their representations are perpendicular to the others and whether the stated solution is truly perpendicular. On the other hand, students of course know that the solution to the problem must fit the definition that distance involves the concept of straightness. This is an example that sometimes student knowledge is only based on definitions that are the initial source and then develop to lead to the right meaning and direction (Herbst et al., 2017; Rahayu & Alghadari, 2019). Furthermore, students' knowledge of geometry is sometimes influenced by their attention to the perceptual display of images that are not certain of their nature. In the citation of the study by Hwang et al. (2009) stated that most children used visual perception to intuitively compare the objects. Whereas the second case is the concept of discipline present as the main concern for making solutions. The concept of discipline is not only contained in the settlement but also mentioned in its interpretation. The representations made by students definitively fulfill their function as solutions and do not conflict with the initial and goal states. It seems that the student conceptualizing the solution through this second case did not shift their focus to objects of thought other than the lines and planes mentioned in the given information. Conceptually, the focus of students in the second case is less than the first case. However, both the solutions made by students through the thought process for the first and second cases, the product of thought is not a class of shape because their attention is not greater on that. Class of shape becomes a product of thought when students state that two diagonal of the plane in a cube are perpendicular to each other as the reason for the solution.

The existence of these two techniques is due to the opportunity for students to find the same distance from different thinking processes. The concept is integrated with different ways and stages (Fauskanger & Bjuland, 2018). Students produce these conceptual tools for constructing, writing, and explaining mathematically significant systems (Biccard, 2018; Dossey, 2017; Lesh & Harel, 2003). Guarino et al. (2004) state that people who solve problems have the same goals but each does something to achieve that goal, but because they do not work together it is appropriate that there are different thinking patterns in the conceptualization process. Furthermore, each person has different expertise and understanding when solving the same problem (Alghadari et al., 2019; Biccard, 2018), due to the work environment, the cases they experience, and the knowledge they build (Hwang et al., 2012), formal and informal knowledge, heuristics, control and belief systems (Dossey, 2017). Each student with his thinking ability creates a diverse design because it suits the needs of the completion process (Biccard, 2018). There are different patterns of thinking to solve the same problem because of conception differences (Yilmaz & Argun, 2017). Therefore knowledge and conception affect students' ability to solve geometry problems (Fauskanger & Bjuland, 2018; Minarti et al., 2018; Yee, 2017). In addition, there are also factors of the problem itself that can lead students to think in different ways so that the approach to solving them is not unique.

At the analysis level, their first answer can be justified but there are problems in other details. It is clear that the other details mentioned by students have nothing to do with the concept of distance and two parallel lines. Their overall answers should be relevant to the problem being solved. In this case, based on students' interpretation that they only integrated a part of the concept when they conceptualized problem solving. The concept is integrated partially and not simultaneously. The fact is that there are concepts that are not always involved in conceptualizing a solution, namely a distance and two parallel lines. Conceptualization is the process of designing to produce concepts in which the application addresses all that is needed (Guarino et al., 2004). Synthesizing ideas is the best way to learn mathematics that is approved by experts (Duru, 2010). When one element is not integrated in the conceptualization process, the solution of the problem is doubtful (Van de Walle et al., 2017). This case is no different from the study findings of Alghadari and Herman (2018) that students who do not correctly solve geometry problems due to the way they combine between concepts so that it affects the completion procedure. On the other hand, there is a quote which states that high school students have great difficulty in communicating the nature of three-dimensional objects that are represented in a two-dimensional format (Herbst et al., 2017; Rahayu & Alghadari, 2019). The nature of three-dimensional objects is an example of the property of shape and also the product of thought at the level of analysis (Van de Walle et al., 2017).

On the problem of geometry level abstraction, one stage was successfully carried out. The three students have also tried to implement the conceptual design for another stage, there is a concept that was integrated into the design experiment, but they have not been successful in implementing it because there are classes and properties of shapes based only on claims. The claim from students is a case that is not in the sense of conception because their perception does not involve concepts as the basic reason for interpreting (Rosilawati & Alghadari, 2018). Students' interpretations of claims do not describe reasons based on rigorous geometry analysis. In the limas geometry element, there are two kinds of claims made by students, namely claims for the class of shapes made by two students and claims for property of shape made by one student to another. The class of shape of a geometry object appears not based on the analysis of the deduction of the properties of shape and its relationship, so we find that the case is a claim that they made (Herbst et al., 2017). While the property of shape appears as a claim because there is an analysis of properties that is skipped by students in the conceptual design stages of problem solving.

The emergence of a claim is the case that the epistemology of the concept of geometry becomes an obstacle for students to solve geometry problems at the level of abstraction. According to the study of Rosilawati and Alghadari (2018), there are epistemological obstacles because students do not identify facts and concepts as a whole. Then, each process after the claim, the three students always involve the Pythagorean theorem in their calculations. The integration of the theorem becomes one fixed direction to solve geometry problems when it involves numerical calculation. The application of the Pythagorean theorem becomes an essential element in the performance of computational geometry. On the other hand, students have not successfully completed the stages of their design because there is a mistake in using the given information. Besides transcribing information from the question, Veloo et al. (2015) have stated that the mistakes most often made by students are due to conceptualization error. These cases occur specifically based on conceptual design with the formation of three parallel stages for one next stage.

Students Inconsistency and Its Implication

This study found that there are stages in students' conceptual designs that lead them to the problem of the problem. The new problem that arises because there is an analysis of properties of geometry has been skipped and the consequence is to add to the direction of the investigation branch. Cases that occur when students discover new problems in their conceptual design are students making claims on objects of thought that are analyzed when they should find a relationship between properties. Students have filled it by assuming and directing their claims for a coherent relationship between properties and the application of the Pythagorean theorem. Xin (2008) states that procedural knowledge involves organizing conceptual knowledge into an action plan. In other words, design and conceptual knowledge have led students towards this procedural knowledge (Fauskanger & Bjuland, 2018). This calculation technique is relatively often used by students in high school because according to Alghadari and Herman (2018) study that the Pythagorean theorem is a geometry concept that is essential for them. However, it should be noted that computation is not modalities of pure abstractive operations and are a lower level of mathematical ability than abstracting (Van de Walle et al., 2017; Yilmaz & Argun, 2017).

The assumption is an indication that students' conceptual knowledge is not strong (Duru, 2010; Minarti et al., 2018), because conceptual knowledge is achieved by the construction of relations between pieces of information (Xin, 2008). The depth of students' knowledge requires the basic knowledge they have (Alghadari et al., 2020; Fauskanger & Bjuland, 2018; Simon, 2017), because conceptualization provides power for integrative work (Dossey, 2017; Nikitina, 2006). An inconsistent performance like this is the impact of students' analysis techniques on geometry objects. The analysis technique is based on the object-oriented approach because visual objects have taken the most focus of student attention in the conceptualization of the solution. In fact, in the case of students designing concepts to solve analytical level geometry problems. In his analysis, students did not elaborate in detail the properties and the relationship between them, and the error was started using the properties from given information. There is a similarity in the problem of students' experience solving problems between the findings of this study and the study by Haryanti (2018), which is the result of them paying less attention to notation. In addition, this consistency problem is also the same as the findings of the study by Marfuah et al. (2016), which is due to abstraction in which the conceptual design that students make in this study has not yet reached the stage of abstracting. This is a resource about students' conceptual and procedural choices to be facilitated in mathematics classrooms (Biccard, 2018), which provides useful input according to students' cognitive challenges and coherent strategies for designing teaching and learning activities that help students to be more competent in mathematics (Veloo et al., 2015). Therefore, there are recommendations listed in Al-shehri et al. (2011) that the level of learning must include the level of integration in the education strategy so that students summarize what is learned to form a comprehensive and deductive picture.

CONCLUSION

Cases are found in the conceptual design of problem solving. First, there are differences in conceptual design techniques. There are two design techniques, namely the coherent entity present as the reason after the solution exists or the main concern for making a solution. The difference in technique is because there is an opportunity for students to integrate concepts in different techniques and students' thinking processes. Second, there is the concept of the problem which is not always involved in the design. The reason is that students conceptualize the initial state, problem, and goal state separately and do not see the problem space as a whole. In addition, the object image has taken most of the focus of student attention so that the analysis technique is based on the object-oriented approach. Third, there are stages of performance that lead students to the problem of the problem. The new problem must be based on rigorous geometry analysis so that the class of shape is deducted based on the properties of shape and its relationship. However, the property of shape appears as a claim because there is an analysis of properties that is skipped over and students make claims on objects of thought when they should find a relationship between properties. Then, students direct their claims for a coherent relationship between properties and the application of the Pythagorean theorem. Even in any scope, the claim is worth questioning. Numerical computing becomes an essential side of geometry analysis while computation is not modalities of pure abstractive operations. Arranging concepts in the structure of student design schemes does not always come up with the right solution. Inconsistent performance is the impact of students' analysis techniques on object geometry. The study results concluded that the performance of students' geometry analysis was consistent with the concept of epistemological problems. In an educational context, this conclusion points to the importance of understanding the epistemology of concepts, the skill of integrating concepts or information more effectively, and developing abstract thinking abilities.

REFERENCES

Al-shehri, M. A., Al-zoubi, S., Bani, M., & Rahman, A. (2011). The Effectiveness of Gifted Students Centers in Developing Geometric Thinking. *Educational Research*, *2*(11), 1676–1684.

Alghadari, F., & Herman, T. (2018). The obstacles of geometric problem-solving on solid with vector and triangle approach. *Journal of Physics: Conference Series*, *1132*(1), 012046. https://doi.org/10.1088/1742-6596/1132/1/012046

Alghadari, F, & Herman, T. (2018). The Obstacles of Geometric Problem-Solving on Solid with Vector and Triangle Approach. *Journal of Physics: Conference Series*, *1132*(1), 012046. https://doi.org/10.1088/1742-6596/1132/1/012046

Alghadari, F, Yuni, Y., & Wulandari, A. (2019). Conceptualization in solving a geometric-function problem: An effective and efficient process. *Journal of Physics: Conf. Series*, 012004. https://doi.org/10.1088/1742-6596/1315/1/012004

Alghadari, Fiki, Herman, T., & Prabawanto, S. (2020). Factors Affecting Senior High School Students to Solve Three-Dimensional Geometry Problems. *International Electronic Journal of Mathematics Education*, *15*(3), em0590. https://doi.org/10.29333/iejme/8234

Biccard, P. (2018). Mathematical sense-making through learner choice. *Pythagoras*, *39*(1), 1–9. https://doi.org/10.4102/pythagoras.v39i1.424

Dossey, J. A. (2017). Problem Solving from a Mathematical Standpoint. In C. Benő & F. Joachim (Eds.), *Educational Research and Innovation: The Nature of Problem Solving Using Research to Inspire 21st Century Learning* (pp. 59–72). OECD Publishing.

Duru, A. (2010). The experimental teaching in some of topics geometry. *Educational Research and Review*, *5*(10), 584–592.

Edmonds, W. A., & Kennedy, T. D. (2016). *An applied guide to research designs: Quantitative, qualitative, and mixed methods*. Sage Publications.

Fauskanger, J., & Bjuland, R. (2018). Deep Learning as Constructed in Mathematics Teachers' Written Discourses. *International Electronic Journal of Mathematics Education*, *13*(3), 149–160. https://doi.org/10.12973/iejme/2705

Guarino, N., Oberle, D., & Staab, S. (2004). What Is an Ontology? In S. Staab & R. Studer (Eds.), *Handbook on Ontologies* (pp. 1–17). Springer Berlin Heidelberg.

Haryanti, S. (2018). Analisis Hasil Assesmen Diagnostik Kemampuan Komunikasi Matematika Siswa Pada Pembelajaran Dimensi Tiga Tipe TPS Kelas X. *Jurnal Karya Pendidikan Matematika*, *5*(2), 76. https://doi.org/10.26714/jkpm.5.2.2018.76-91

Herbst, P., Fujita, T., Halverscheid, S., & Weiss, M. (2017). *The Learning and Teaching of Geometry in Secondary Schools: A Modeling Perspective*. Routledge.

Hock, T. T., Tarmizi, R. A., Aida, A. S., & Ayub, A. F. (2015). Understanding the primary school students' van Hiele levels of geometry thinking in learning shapes and spaces: A Q-methodology. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(4), 793–802. https://doi.org/10.12973/eurasia.2015.1439a

Hwang, G. J., Panjaburee, P., Triampo, W., & Shih, B. Y. (2012). A group decision approach to developing concept-effect models for diagnosing student learning problems in mathematics. *British Journal of Educational Technology*, *44*(3), 453–468. https://doi.org/10.1111/j.1467-8535.2012.01319.x

Hwang, W., Su, J., Huang, Y., & Dong, J. (2009). A Study of Multi-representation of Geometry Problem Solving with Virtual Manipulatives and Whiteboard System. *Educational Technology & Society*, *12*(3), 229–247.

Isnawan, M. G., Suryadi, D., & Turmudi, T. (2022). How secondary students develop the meaning of fractions? A hermeneutic phenomenological study. *Beta: Jurnal Tadris Matematika*, *15*(1), 1–19. https://doi.org/10.20414/betajtm.v15i1.496

Kemdikbud. (2023). Rapor Pendidikan Indonesia tahun 2023. In *Rapor Pendidikan*. https://raporpendidikan.kemdikbud.go.id/

Kemdikbudristek. (2022). *Capaian Pembelajaran Mata Pelajaran Matematika Fase A - Fase F untuk SD/MI/Program Paket A, SMP/MTs/Program Paket B, dan SMA/MA/Program Paket C*. Badan Standar, Kurikulum, dan Asesmen Pendidikan – Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi.

Lesh, R., & Harel, G. (2003). Problem Solving, Modeling, and Local Conceptual Development. *Mathematical Thinking and Learning*, *5*(2), 157–189. https://doi.org/10.1207/s15327833mtl0502&3_03

Lin, F.-L., Yang, K.-L., & Chen, C.-Y. (2004). The Features and Relationships of Reasoning, Proving and Understanding Proof in Number Patterns. *International Journal of Science and Mathematics Education*, *2*(2), 227–256. https://doi.org/10.1007/s10763-004-3413-z

Mammana, M. F., Micale, B., & Pennisi, M. (2012). Analogy and dynamic geometry system used to introduce three-dimensional geometry. *International Journal of Mathematical Education in Science and Technology*, *43*(6), 818–830. https://doi.org/10.1080/0020739X.2012.662286

Marfuah, S., Zulkardi, Z., & Aisyah, N. (2016). Pengembangan Media Pembelajaran Menggunakan Powerpoint Disertai Visual Basic for Application Materi Jarak Pada Bangun Ruang Kelas X. *Jurnal Gantang*, *1*(1), 45–53. https://doi.org/10.31629/jg.v1i1.5

Minarti, E. D., Wahyudin, & Alghadari, F. (2018). Student's conceptions and geometry problem-solving of the distance in cube. *Journal of Physics: Conference Series*, *1132*(1), 012033. https://doi.org/10.1088/1742-6596/1132/1/012033

Nikitina, S. (2002). Three strategies for interdisciplinary teaching: Contextualizing, conceptualizing, and problem-solving. In *Interdisciplinary Studies Project Zero*. https://doi.org/10.1080/00220270500422632

Noor, N. A., & Alghadari, F. (2021a). Conceptual technique for comparison figures by geometric thinking in analysis level. *Journal for the Mathematics Education and Teaching Practices*, *2*(1), 1–8.

Noor, N. A., & Alghadari, F. (2021b). Case of Actualizing Geometry Knowledge in Abstraction Thinking Level for Constructing a Figure. *International Journal of Educational Studies in Mathematics*, *8*(1), 16–26. https://doi.org/10.17278/ijesim.797749

Nur, C. I., Armiznah, D., & Rismawati, R. (2023). Penggunaan Media GeoGebra dalam Meningkatkan Hasil Belajar Siswa pada Materi Geometri Ruang Kelas XII IPS. *Musamus Journal of Mathematics Education*, *5*(2), 76–81. https://doi.org/10.35724/musamus journal of mathematics education.v5i2.5291

Nurhikmayati, I., Jatisunda, M. G., & Ratnawulan, N. (2022). The Practice of Reflection Based on Didactical Design Research: An Analysis of the Geometry Transformation Material. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, *6*(3), 565. https://doi.org/10.31764/jtam.v6i3.8441

OECD. (2019). *PISA 2018 Results (Volume I)*. OECD. https://doi.org/10.1787/5f07c754-en

Paryatun, S., Usodo, B., & Saputro, D. R. S. (2016). Eksperimentasi Pembelajaran Matematika Dengan Model Problem Based Learning (PBL) Dan Learning Cycle 5E Dengan Pendekatan Saintifik pada Materi Dimensi Tiga Ditinjau Dari Kemandirian Belajar Siswa Kelas X SMA Negeri di Kabupaten Sukoharjo. *Jurnal Elektronik Pembelajaran Matematika Vol.4, 4*(3), 341–351.

Pittalis, M., & Christou, C. (2010). Types of reasoning in 3D geometry thinking and their relation with spatial ability. *Educational Studies in Mathematics*, *75*(2), 191–212. https://doi.org/10.1007/s10649-010-9251-8

Pusat Penilaian Pendidikan. (2019). *Daya Serap*. Persentase Siswa Yang Menjawab Benar Tahun Pelajaran 2018/2019. https://hasilun.puspendik.kemdikbud.go.id/#2019!sma daya_serap!99&99&999!a&03&T&T&1&!1!&

Rahayu, T., & Alghadari, F. (2019). Identitas Bayangan Konsep Limas: Analisis Terhadap Konsepsi Matematis Siswa. *Inomatika: Jurnal Inovasi Matematika*, *1*(1), 7–20. https://doi.org/10.35438/inomatika.v1i1.134

Rawuh, R. (2014). *Geometri* (5th ed.). Universitas Terbuka.

Retta, A. M., & Fitriasari, P. (2022). Pengembangan Lembar Kerja Siswa Berbantuan Geogebra pada Materi Dimensi Tiga untuk Siswa SMA. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *11*(2), 819. https://doi.org/10.24127/ajpm.v11i2.4249

Romenskyy, M., Herbert-Read, J. E., Ioannou, C. C., Szorkovszky, A., Ward, A. J. W., & Sumpter, D. J. T. (2020). Quantifying the structure and dynamics of fish shoals under predation threat in three dimensions. *Behavioral Ecology*, *31*(2), 311–321. https://doi.org/10.1093/beheco/arz197

Rosilawati, R., & Alghadari, F. (2018). Konsepsi siswa pada suatu bentuk bangun ruang terkait dengan rusuk dan diagonal sisi. *Prisma: Jurnal Pendidikan Matematika*, *7*(2), 164–176. https://doi.org/10.35194/jp.v7i2.459

Sahaf, A., Abdoli, A., & Mohamadzadeh Moghaddam, A. (2021). Calculating TIN-Based 3D Sight Distance and Its Evaluation on Drivers Free-Flow Speed. *Shock and Vibration*, *2021*, 1–7. https://doi.org/10.1155/2021/1337540

Simon, M. A. (2017). Explicating mathematical concept and mathematicalconception as theoretical constructs for mathematics education research. *Educational Studies in Mathematics*, *94*(2), 117–137. https://doi.org/10.1007/s10649-016-9728-1

Suwa, M., Gero, J. S., & Purcell, T. a. (1998). The Roles of Sketches in Early Conceptual Design Processes. *Proceedings of the Twentieth Annual Meeting of the Cognitive Science Society*, *1992*, 1043–1048.

Van de Walle, J. ., Karp, K. ., & Bay-Williams, J. . (2017). *Elementary and Middle School Mathematics: Teaching Developmentally* (M. Fossel, M. Feliberty, L. Bishop, & et al (eds.); 9th ed.). Pearson Education.

Veloo, A., Krishnasamy, H. N., & Wan Abdullah, W. S. (2015). Types of student errors in mathematical symbols, graphs and problem-solving. *Asian Social Science*, *11*(15), 324–334. https://doi.org/10.5539/ass.v11n15p324

Ward, T. B. (2012). Problem Solving. In M. D. Mumford (Ed.), *Handbook of Organizational Creativity* (pp. 169–188). Academic Press.

Wu, Z., Huang, H., Zhao, G., & Liu, J. (2023). TMR-Array-Based Pipeline Location Method and Its Realization. *Sustainability*, *15*(12), 9816. https://doi.org/10.3390/su15129816

Xin, Y. P. (2008). The effect of schema-based instruction in solving mathematics word problems: An emphasis on prealgebraic conceptualization of multiplicative relations. *Journal for Research in Mathematics Education*, *39*(5), 526–551. https://doi.org/10.5951/jresematheduc.39.5.0526

Yee, S. P. (2017). Students' and Teachers' Conceptual Metaphors for Mathematical Problem Solving. *School Science and Mathematics*, *117*(3–4), 146–157. https://doi.org/10.1111/ssm.12217

Yilmaz, R., & Argun, Z. (2017). Role of Visualization in Mathematical Abstraction: The Case of Congruence Concept. *International Journal of Education in Mathematics, Science and Technology*, *6*(1), 41–57. https://doi.org/10.18404/ijemst.328337

Yuliani, R. E., Heru, H., & Sari, E. L. (2021). Pengaruh Penggunaan Aplikasi Geogebra Berbasis TPACK terhadap Hasil Belajar Matematika pada Materi Bangun Ruang di SMA Negeri 19 Palembang. *Publikasi Penelitian Terapan Dan Kebijakan*, *4*(1), 12–17. https://doi.org/10.46774/pptk.v4i1.337